Supply-Based Models of Suspended Sediment Transport in Streams

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Sediment supplies and stream discharge together determine the patterns, over time, of suspended sediment loads in small streams. Most of the uncertainty in empirical streamflow-sediment relationships can be attributed to changing supplies. Our transport model utilizes a power function of the form $C = aQ^b$, where C and Q are sediment concentration and stream discharge, respectively. This expression was augmented with a variable S representing sediment storage in the channel system. The resulting supply-based model was calibrated to concentration and streamflow time series data from four storm events in a small forested watershed in coastal Oregon. We also calibrated the model to data from a controlled reservoir release in Utah, during which streamflow was held constant for an extended period. In all cases the supply-based model followed observed concentration time series more accurately than did a transport model based on Q alone. We further enhanced performance of the supply-based model by distributing sediment supplies S among several compartments which were accessed at different levels of stream discharge. Both the single-compartment and distributed models demonstrate that a knowledge of sediment supplies can improve predictions of suspended sediment concentrations during storm runoff.

INTRODUCTION

Sediment yields from small watersheds are difficult to model and predict. The sediment load of a small stream typically shows considerable variability over time, as it responds in a highly sensitive, nonlinear fashion to changes in streamflow and sediment availability. Unfortunately, the set of processes in stream systems which link sediment sources and runoff to sediment transport have not yet been well identified, much less quantified [*Wolman*, 1977]. Much of the difficulty lies in merely observing sediment transport, since the majority of the sediment load is usually carried during brief, infrequent periods of high runoff.

In view of these problems, it is not surprising that the most practical sediment transport models continue to be empirical relations between sediment load and streamflow. The simplest model of this type is the familiar sediment transport curve or rating curve

$$C = aQ^b \qquad (1)$$

in which Q is stream discharge and C is either suspended sediment concentration or yield. Values of a and b for a particular stream are determined from data via a linear regression between (log C) and (log Q). Equation (1) is usually combined with a streamflow duration curve to estimate mean annual yield [*Piest and Miller*, 1975].

The simple transport curve has been extended in several directions. *Guy* [1964] and many others have studied multivariate forms of (1), with factors such as rainfall and time of event occurrence as possible independent variables. One stochastic extension of the transport curve views Q as a stochastic process in order to estimate the temporal variability of sediment yield [*VanSickle*, 1982]. Another stochastic approach involves Box-Jenkins type transfer function models, in which the present value of C depends on past values of C, as well as present and past values of Q [*Sharma et al.*, 1979].

These extended models have all tried to improve the low accuracy seen in most applications of (1). A stream in the Oregon Coast Range provides a typical example (Figure 1)

where observed suspended sediment concentrations vary up to an order of magnitude at a given discharge level. Most of the scatter about the regression line in the figure is probably due to changing supplies of available sediment. Although sediment supply is generally recognized as the single most important factor (other than streamflow) which determines watershed sediment yield patterns over time, sediment storage or supply remains an elusive variable, difficult to measure or to model [*Wolman*, 1977]. Studies of annual sediment budgets have recently begun to include direct, quantitative estimates of sediment sources [*Dietrich and Dunne*, 1978; *Kelsey*, 1980]. At present, however, estimates of every term in the budget equation are not yet possible within the time frame of individual runoff events.

The qualitative effects of sediment supply on sediment concentrations and yields have often been described. During a single runoff event, concentrations C at a specific discharge level Q usually decrease with time, owing to supply depletion. When observed values of Q and C are plotted with time as a parameter, over the course of a single storm event, a hysteresis loop often results. These hysteresis loops have been described for streams of all sizes [*Leopold et al.*, 1964; *Shen*, 1971; *Walling*, 1977; *Paustian and Beschta*, 1979; *Whitfield and Schreier*, 1981]. A similar effect is seen on a seasonal time scale; concentrations at a given discharge nearly always decrease as the runoff season progresses and sediments are flushed from the watershed and/or stream system [*Leopold et al.*, 1964; *Piest and Miller*, 1975; *Nanson*, 1974; *Beschta*, 1978].

The phenomena of seasonal decline and storm hysteresis are apparent in most sampled time series of C and Q. In this paper we model those time series by adding a new variable to (1). The new variable $S(t)$ represents sediment supply, and the modified form of (1) expresses the effects of supply depletion on sediment concentrations. We present model applications for which we have no direct estimates of $S(t)$; hopefully, estimates of $S(t)$ will soon be practical. Sediment inputs to streams and in-channel storage sites are just beginning to be identified and quantified as part of larger sediment budget studies [*Swanson et al.*, 1982].

We applied the supply-based model to data from an undisturbed watershed in the Oregon Coast Range and also to data from a controlled reservoir release in central Utah. Model analyses and simulations were performed, with the goals of observing model supply dynamics and of comparing model
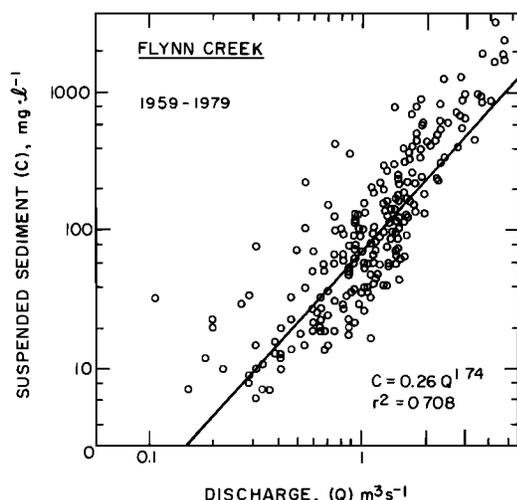
Fig. 1. Sediment transport curve for Flynn Creek, based on instantaneous values from 24 storms having peak flows $\geq 1.1 \text{ m}^3 \text{ s}^{-1}$.

sediment concentrations with those predicted by (1). Our approach is that of 'grey box' modeling [*Pickup*, 1981]; we try to represent the main processes and feedbacks of the sediment delivery system, with only a few variables, parameters, and equations. Recently, *Dietrich et al.* [1982] have proposed grey box type models for routing sediment through entire basins, but the models have not yet been applied to real data. The strategy of these basin models and of our supply-based model is simple frugality; models with few parameters can be reliably calibrated and validated by limited data sets. The simplicity of the model structure helps to compensate for our inability to measure a key model variable, sediment supply.

Modeling Sediment Outputs From a Single Storage Compartment

We began by assuming that the total amount of sediment stored upstream of a sampling location at time t can be lumped into a single storage variable $S(t)$. The supply $S(t)$ is assumed to be suspendable material stored within the stream channel. For the model applications discussed here, the largest storm events had peak flows with return periods of about 2 years, and they peaked at, or slightly above, bankful stage [*Beschta*, 1981].

In the model, $S(t)$ and sediment transport dynamics follow a scenario (Figure 2) which is generally applicable to streams draining lower-elevation (<1200 m) forested watersheds in the Pacific Northwest. At the start of the autumn rainy season, $S(t)$ is assumed to be at an initial maximum, S_0. The supply is periodically depleted through the fall and winter by a sequence of brief (generally ≤ 72 hours), distinct runoff events due to rainstorms. Figure 2 illustrates changes in $S(t)$, suspended sediment concentration $C(t)$, and discharge $Q(t)$ during two events. Between storms, flow is greatly reduced, sediment concentration is very low, and net transport is negligible [*Van-Sickle*, 1981]; the model is not operated during these intervals.

During a runoff event, sediment concentration is modeled with the sediment transport curve (1) modified to include a supply depletion or washout function:

$$C(t) = aQ(t)^b \cdot g[S(t)] \tag{2}$$

The washout function $g(S)$ expresses the relative change in concentration due to changes in available sediment.

As S decreases during a single event or seasonally, con-

centrations also decrease. Thus $g(S)$ should be positive valued with $dg/dS \geq 0$. We will show later that $g(S)$ has an exponential, rather than a linear, nature. However, its exact form is probably not critical. We chose the washout function to be

$$g(S) = p \cdot \exp\left[r \frac{S}{S_0} \right] \tag{3}$$

Equation (3) does not satisfy the intuitive requirement that $g(0) = 0$, but in practice, the parameter p is small and $g(S)$ can be artificially set to zero for $S = 0$ without disrupting the model dynamics of $C(t)$ or $S(t)$. Both p and r are dimensionless, empirically determined parameters.

In order to use (2), we need to keep track of the supply, $S(t)$. The only losses from the storage compartment are assumed to be due to sediment transport, and the flux past the sampling point is simply $Q \cdot C$. Hence the complete model is

$$\frac{dS(t)}{dt} = -Q(t) \cdot C(t) \tag{4}$$

$$C(t) = aQ(t)^b \cdot p \cdot \exp\left[r \frac{S(t)}{S_0} \right] \tag{5}$$

In the model, $C(t)$ has units of milligrams per liter, equivalent to grams per cubic meter, and $Q(t)$ is in cubic meters per second. $S(t)$ then has units of grams, but we report both supply and sediment yield in metric tons (tonnes).

We do not have definitive physical interpretations for the parameters in (5), because they summarize the multitude of factors which determine sediment transport rates. Roughly speaking, however, the parameters a and b may be associated with characteristics of the channel system, such as hydraulic geometry, channel morphology, gradient, etc., which determine transport rates at a given discharge and level of sediment availability. The parameter r can be interpreted as an index of sediment availability. For a large value of r, concentrations are
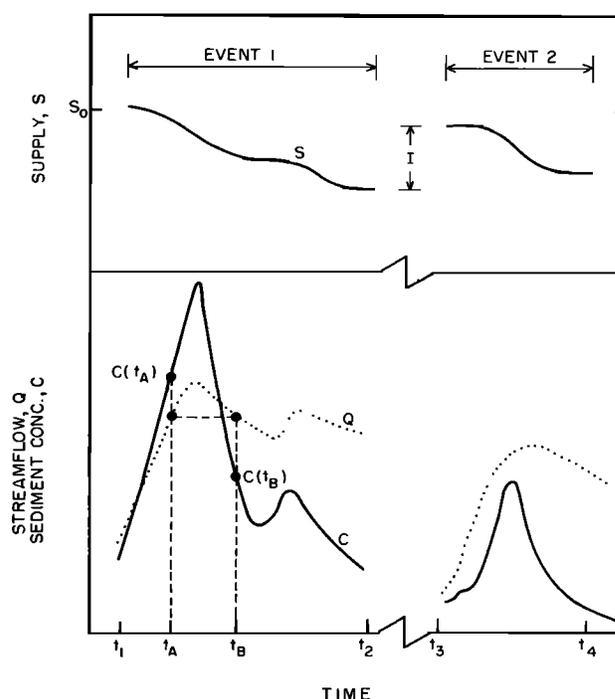

Fig. 2. Idealized variations of sediment supply S, sediment concentration C, and streamflow Q during storm events.

sensitive to small decreases in $S(t)$; the model sediment supply has relatively low availability. Thus r is likely to be a function of the bed composition and of the overall effectiveness of storage sites in retaining sediment.

Equations (4) and (5) apply over any time interval (t_1, t_2) spanned by a single runoff event. For their solution, an initial supply $S(t_1)$ and a storm hydrograph $Q(t)$, for $t_1 \le t \le t_2$, must be specified. If t_1 represents the start of the first fall storm, then $S(t_1) = S_0$. If $Q(t)$ can be integrated, then (4) can be solved analytically, via separation of variables. Here we present numerical solutions, based on sampled time series for $Q(t)$.

Because of the nonpoint nature of sediment sources distributed throughout a channel system, discrete sediment storage locations are not easily identifiable. In-channel supplies during high flows might include the release of suspended sediments from riffles undergoing scour, erosion and sloughing of bank sediments, and the scouring of deposits in pools. However, at least in the Pacific Northwest, mass soil erosion (landslides, soil creep, and debris avalanches) represents the ultimate source of sediments for many mountain streams. Sediment inputs to storage are as complex in space and time as are the storages themselves and just as difficult to quantify. For now, we are forced to model sediment inputs in the same, almost abstract, fashion as we define $S(t)$. We assume that a sediment input I in the model occurs as a lump sum of material added between storms (Figure 2). In practice, the input merely resets $S(t)$ between events.

Model time series of $C(t)$ can be compared directly with data, but we have no observations for $S(t)$. However, losses from supply, i.e., sediment yields, can be directly calculated from C and Q data. Let $Y(t, t_1)$ be the cumulative yield over the period (t_1, t), where $t \ge t_1$. During the course of a single event, a useful auxiliary equation is

$$Y(t, t_1) = \int_{t_1}^{t} C \cdot Q \cdot dt = S(t_1) - S(t) \qquad (6)$$

Several researchers have suggested that sediment transport be modeled as a function of sediment supplies. For example, *Piest and Miller* [1975] suggest that the coefficient a in (1) can be viewed as an index of relative erodibility. In addition, *Nolan and Janda* [1981] find evidence in their data that increasing sediment availability tends to increase the intercept, but not the slope, of log-log regressions of C on Q. They suggest that a in (1) reflects sediment supplies, while the b exponent empirically summarizes the sediment delivery mechanisms of a particular watershed. These suggestions are similar to our tentative interpretations of the parameters in (5). Negev carried this approach further in his 1967 model, as described by *Linsley et al.* [1975]. The core of the larger Negev model is a sediment transport expression like (2) with $g(S) = K_1 + K_2 S$. The process model of *Li et al.* [1976] also contains a compartment for sediment supply. Possibly because of their complexity, the models of Negev and of *Li et al.* [1976] have not, to our knowledge, been used to explore the supply-transport coupling.

The form of (2) and (5) is well suited to studying the variability among sediment concentrations which are observed at the same streamflow levels. As shown in Figure 2, let t_A and t_B be two different times for which $Q(t_A) = Q(t_B)$. The concentration ratio $R_c(t_B, t_A)$ can then be defined as

$$R_c(t_B, t_A) = \frac{C(t_B)}{C(t_A)} \qquad (7)$$

If $C(t)$ is given by the model of (5), then R_c satisfies the expression

$$R_c(t_B, t_A) = \exp\left[\frac{r}{S_0}(S(t_B) - S(t_A))\right] \qquad (8)$$

Notice that R_c is independent of streamflow and depends only on the net change in supply over the interval (t_A, t_B). This same feature is still true for the more general concentration model (2). In Figure 2, t_A and t_B are times on the rising limb and falling limb, respectively, of a single event. In this case, R_c expresses the magnitude of the storm hysteresis effect.

One could also identify a particular flow at time t_A during the rising limb of the first fall storm hydrograph and a corresponding flow at time t_B during a storm hydrograph late in the runoff season. In this case, R_c would measure the seasonal flushing of sediment supplies. Informal estimates of R_c by *Guy* [1964] and *Piest and Miller* [1975] suggest values of R_c as low as 0.1 to 0.05 for individual storms, while *Nanson* [1974], *Piest and Miller* [1975], and *Loughran* [1976] estimate an R_c value as low as 0.02 to 0.005 over the course of a single year.

The ratio R_c is useful in parameter estimation. Returning to the scenario of Figure 2, with t_A and t_B on the rising and falling hydrograph limbs, we combined the definition of sediment yield (6) with (8), and following logarithmic transformation, we obtained

$$\log_e [R_c(t_B, t_A)] = -\frac{r}{S_0} Y(t_B, t_A) \qquad (9)$$

This expression allows direct use of sample data to estimate washout parameters. If several pairs $(C(t_A), C(t_B))$ at the same discharges can be observed during a single event, then (9) gives a regression estimate of r/S_0.

FLYNN CREEK SIMULATIONS

We applied (4) and (5) to sampled time series of $Q(t)$ and $C(t)$ for four storm events from the Flynn Creek watershed in the Oregon Coast Range. This undisturbed 202-ha watershed has moderately steep (25–40% grade) hillslopes which are forested with Douglas fir (*Pseudotsuga menziesii*) and red alder (*Alnus rubra*). The sediment parent materials are uplifted sedimentary rocks, and soil is delivered to the channel primarily by mass erosion. *Brown and Krygier* [1971] describe the Flynn Creek watershed in more detail.

During rainfall periods, most water is routed as subsurface flow to the channel system. Upstream of the sampling station, the third-order Flynn Creek channel varies from 3 to 5 m in width; bed material consists predominately of medium to coarse sand and fine gravel (0.25–8.0 mm), armored with fine to coarse gravel (4–32 mm). Channel geometry is largely influenced by large organic debris (fallen logs) and the root systems of the existing forest vegetation. The large organic debris affects local channel hydraulics and creates storage sites for suspended and bedload sediment. During the winter, most precipitation results from long-duration and low-intensity frontal storms that move inland from the Pacific Ocean. Within these major frontal storms, periods of moderate to high rainfall intensities generate runoff events in which most sediment is transported.

Streamflow and sediment concentrations in Flynn Creek were closely monitored between 1959 and 1973 as part of the Alsea Watershed study [*Brown and Krygier*, 1971; *Harris*, 1977]. Based on these data, we have used statistical analyses to describe long-term patterns of monthly and annual sediment
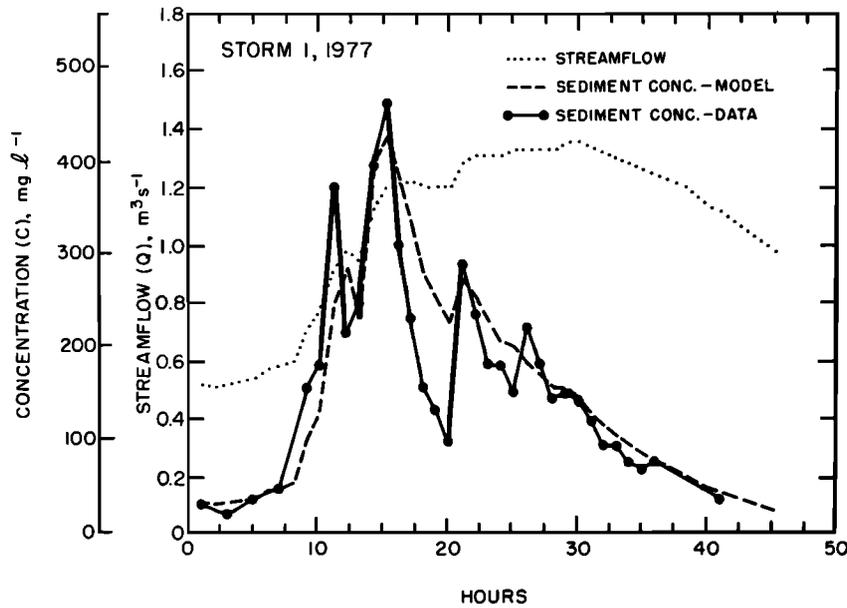
Fig. 3. Data versus calibrated, single-supply model concentrations. Flynn Creek, first autumn storm, 1977.

yield from the watershed [Beschta, 1978; VanSickle, 1981, 1982].

In addition, transport dynamics have been studied in Flynn Creek on the much shorter time scale of individual runoff events [Beschta et al., 1981a]. Three of the four events modeled here occurred sequentially in November and December 1977 with the initial event being the first sizable fall runoff. The fourth event was the first fall storm of 1979. Beschta [1981] describes the four events and their sediment yields in more detail.

During the runoff events, suspended sediment concentration and stream discharge were measured hourly. To estimate cumulative yield from the data, via (6), we filled gaps in the $C(t)$ record using linear interpolation. Simulations of (4) employed a Runge-Kutta method with a variable time step of ≤ 1 hour. The forcing function $Q(t)$ was calculated by linear interpolation between hourly values of the observed Q time series.

A set of model simulations was used to calibrate the model to fit observed (Q, C) time series. The calibration process also served as an informal sensitivity analysis. Model performance was measured by the time-averaged absolute difference between model and data values of $C(t)$. For practical purposes, sediment yield errors may be more important than concentration errors. However, it is not difficult to show that the supply-based model can be calibrated to give any desired sediment yield for a single runoff event, so the yield performance was hardly a fair measure. The supply model concentration errors E^S were compared with errors E^T produced by the simple transport curve (1). Transport curve errors E^T were calculated by fitting (1) to the data on a storm-by-storm basis; different values of a and b were used for each event.

Calibration of the supply model was more involved. Equation (5) appears to have five free parameters: a, b, p, r, and S_0. We reduced this number to four by adjusting p so that $g(S_0) = 1$, once S_0 and r had been chosen. This restriction also let us interpret the transport curve parameters (a, b) as reflecting conditions of full supply; with $g(S_0) = 1$, the concentration is simply $C = aQ^b$. Ideally, then, a and b could be determined from (C, Q) data taken during the rising limb of the first fall

storm hydrograph of 1977. This procedure gave us initial estimates for a and b of 1.5 and 3.2, respectively.

We also used data from the same event to estimate r/S_0. Equation (9) was applied to seven different discharge levels on the rising and falling hydrograph limbs from the first 1977 event, and the average value of $r/S_0 = 0.102$ tonne^{-1} was used as an initial estimate. One parameter, S_0, was set somewhat arbitrarily. We reasoned that the initial supply should exceed a typical annual sediment yield by a considerable margin. Consequently, we chose $S_0 = 303$ tonnes, which corresponded to an annual yield with an approximate 5-year return period, based on the 1959–1973 record [VanSickle, 1981]. This amount of sediment represents approximately 23 kg of suspended sediment per meter of channel length if it were distributed uniformly throughout the first-, second-, and third-order channels on the Flynn Creek watershed. The corresponding initial value of r was 30.9.

The next calibration step consisted of 'fine tuning' the model for the first 1977 storm event. Holding S_0 fixed, we varied r, a, and b by trial and error until the fit shown in Figure 3 was achieved. The associated model errors E^S for this and other events are listed in Table 1. Model results, illustrated in Figure 3, used values of $r = 26.6$, $a = 0.85$, and $b = 3.75$. Thus the final parameter values were close to their initial estimates.

Even though we do not yet have clear physical interpretations for a, b, and r, we expect the three parameters should remain fairly constant over time for a given stream, barring the occurrence of large-scale geomorphic events. Thus, a, b, and r were held fixed at the above values for model calibration with the other three Flynn Creek storms.

For the two remaining 1977 events, only sediment inputs between storms were adjusted to achieve the fits shown in Figure 4. Between storms 1 and 2 we added $I = 19.5$ tonnes to $S(t)$, and between storms 2 and 3, $I = 7.8$ tonnes were added to $S(t)$. It is interesting that the input between storms 1 and 2 was about 75% as large as the total observed sediment yield from storm 1. The value of I between events 2 and 3 was about equal to the yield from storm 2. The model reflects the fact that concentrations on the falling limb of a storm hydrograph were

TABLE 1. Concentration Errors for Calibrated Models

Storms	N	Single-Compartment Model, Equations (4) and (5)		Distributed Model, Equation (10)	
		E^S, mg l^{-1}	E^S/E^T	E^S, mg l^{-1}	E^S/E^T
1 (1977)	34	35.8	0.46	43.0	0.55
2 (1977)	15	39.9	0.95	21.6	0.51
3 (1977)	21	24.5	0.52	17.4	0.37
1 (1979)	52	98.7	0.78	88.2	0.70

Supply model error E^S versus transport curve error E^T, where $E = (1/N)\Sigma_{i=1}^{N}|C_i(\text{data}) - C_i(\text{model})|$.

consistently lower than concentrations at the same discharges on the rising limb of the next storm hydrograph. If these differences are indeed due only to changing supplies, then some new material must have become available between storm events.

For the 1979 event, we retained the 1977 parameter values and reset $S(t_0) = S_0$, since the 1979 storm was the first of the season. The result was a serious underprediction of observed concentration levels, over the course of the entire hydrograph. As before, we tried calibrating the model through addition of inputs prior to the event simulation, with little success.

The source of the poor model performance was discovered when we compared R_c and cumulative sediment yields from the data for the first storms of 1977 and 1979. During the course of the 1979 event, values of R_c were similar to those of storm 1, 1977, but the associated sediment yields were about 3 times larger. In other words, the same relative decrease in concentration was accompanied by a much larger loss from storage in the 1979 event, as compared with 1977. This difference may have been due to an increased availability of the sediment supply in 1979; the supply appeared to suffer a greater depletion before forcing concentrations to decrease. Accordingly, we again set $S(t_0) = S_0$ and varied the availability parameter r to produce the fit seen in Figure 5. That simulation used $r = 9.7$, with all other parameters taking their 1977 values.

To summarize, we note that model time series for $C(t)$ agreed fairly well with observed values over the course of four events (Figures 3-5). From a purely quantitative standpoint, the supply-based model appeared to perform slightly better than the conventional transport curve, as Table 1 shows. Values of

E^S/E^T are less than 1 for all four events. In addition, the supply model's better fit was achieved with a total of seven parameter estimates for the four events (a, b, S_0, r, followed by two values of I for 1977 plus a value of r for 1979), compared with eight estimates (new values of a and b for each event) for the simple transport curve. However, these differences could probably not be supported statistically.

A Distributed Supply Model

The least realistic feature of (4) and (5) is the lumping of all sediment supplies into a single variable, $S(t)$. For this reason, we explored a model in which the total supply of sediment is distributed among several model storage compartments. In the distributed model, the magnitude of Q determines the set of storage compartments from which sediment may be removed. We assume that, as streamflow increases, sediment is removed from an increasing number of storage sites, and each site is depleted at a rate dependent on supply and discharge.

The model structure is illustrated in Figure 6a. We assign a set of fixed discharge levels, Q_0, Q_1, $\cdots$, Q_N. The level Q_0 is set to be 0 m^3 s^{-1}, and Q_N is taken to be a flow considerably larger than the peaks of the four Flynn Creek events. The levels Q_i, in turn, define a set of N storage compartments, with the ith compartment containing an amount $S_i(t)$ of sediment. The sediment in the ith compartment consists of all material which will be transported only when stream discharge exceeds Q_{i-1}, for $i = 1, 2, \cdots, N$.

As Figure 6a suggests, $S_i(t)$ includes sediment which is stored in the bank between the stage heights which correspond to Q_{i-1} and to Q_i. However, other sources may also be repre-
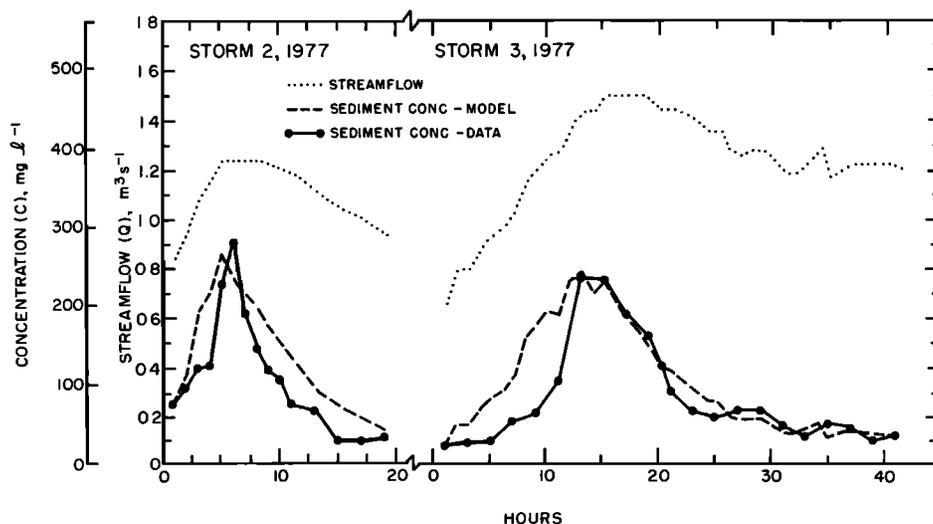

Fig. 4. Data versus calibrated, single-supply model concentrations Flynn Creek, storms 2 and 3, 1977.
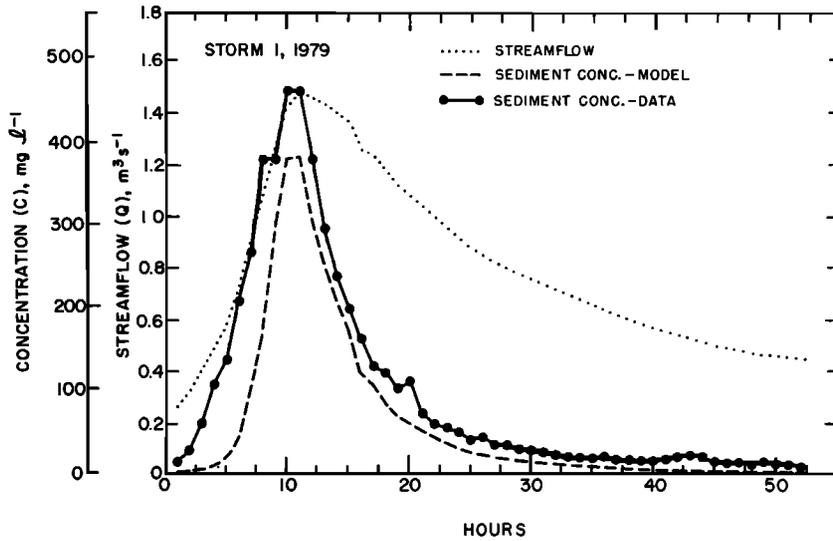
Fig. 5. Data versus calibrated, single-supply model concentrations. Flynn Creek, first autumn storm, 1979.

sented by $S_i(t)$. For example, $S_i(t)$ would include material in the bed which was newly suspended by the increase in turbulence or shear stress at flows greater than Q_{i-1}. Similarly, a discharge greater than Q_{i-1} might be required to release a log or other obstruction in the bed. Sediment stored behind the obstruction would also be considered part of $S_i(t)$. In addition, the headward expansion of the channel system (with a corresponding increase in the capability of the stream to access additional sources of sediment) is also implied in our conceptualization of the distributed supply model.

During a model storm event, the supply of the ith compartment decreases whenever $Q(t) \geq Q_{i-1}$. If $Q(t) < Q_{i-1}$, then the discharge is too low to access the ith compartment, and $S_i(t)$ stays constant. When a compartment is accessed, the sediment

removal rate is again the product of a transport curve and a washout function. These assumptions lead to the model equations for $i = 1, 2, \cdots, N$:

$$\frac{dS_i(t)}{dt} = -a \cdot Q(t)^b \cdot p \cdot \exp\left[r \frac{S_i(t)}{S_i(t_0)} \right] \qquad Q(t) \geq Q_{i-1}$$
$$= 0 \qquad\qquad\qquad\qquad\qquad Q(t) < Q_{i-1}$$
(10)

The parameters in (10) have the same roles as those in (4) and (5), but they are not mathematically equivalent. As with the previous model, inputs to the compartments (I_i, where $i = 1, 2, \cdots, N$) are added between storms and serve to reset the $S_i(t)$ values for a subsequent event.
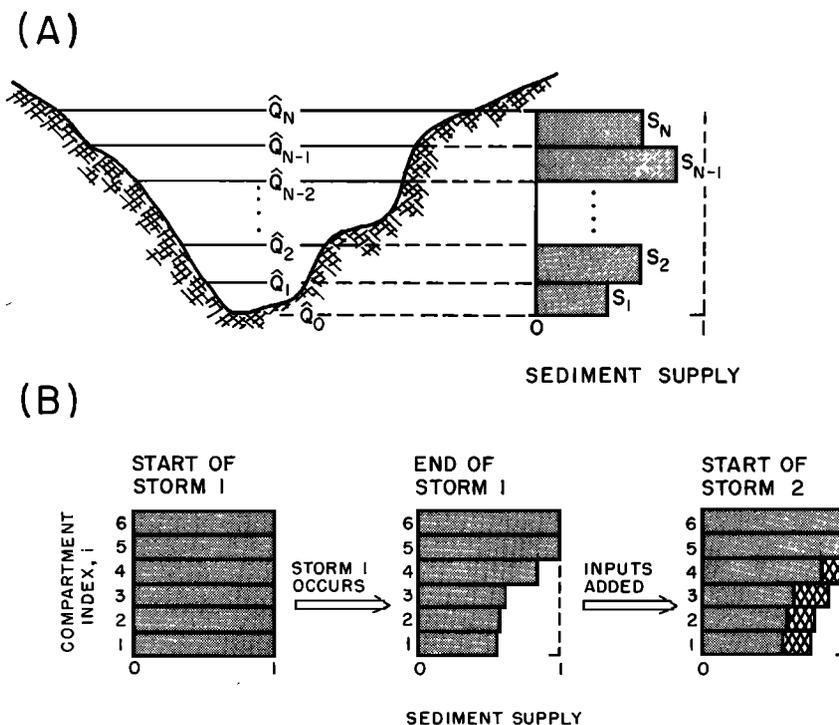

Fig. 6. The distributed supply model. (a) Supply distribution as defined by discharge level. Sediment supply is shown normalized to its initial value $S_i(t_0)$. (b) Model supply distribution before and after storm 1, 1977.
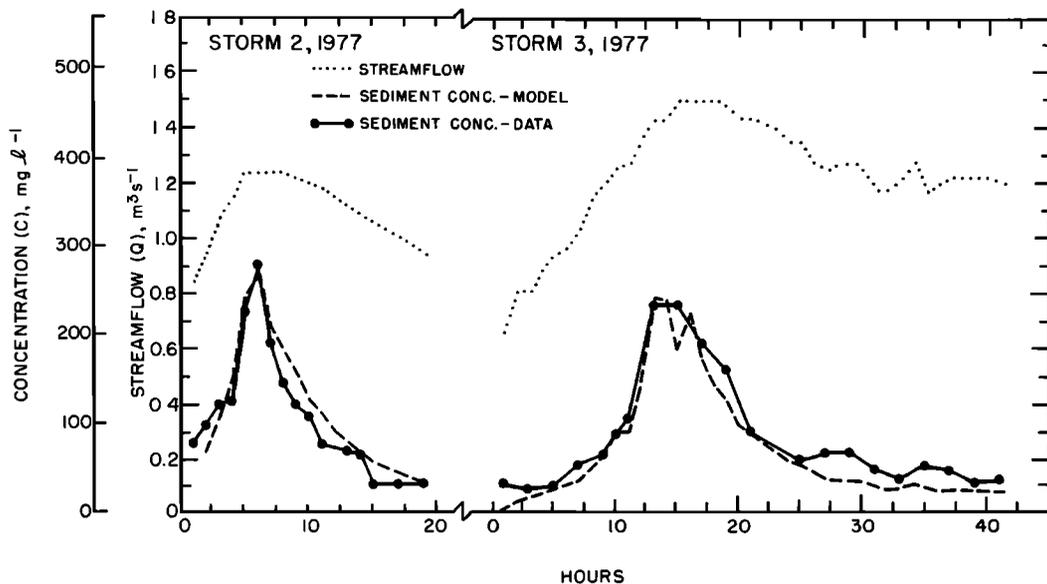
Fig. 7. Data versus calibrated, distributed model concentrations. Storms 2 and 3, 1977.

Total sediment storage in the distributed model is represented by

$$S_T(t) = \sum_{i=1}^{N} S_i(t) \qquad (11)$$

The total transport flux is the sum of losses from all compartments:

$$T(t) = - \sum_{i=1}^{N} \frac{dS_i(t)}{dt} \qquad (12)$$

Sediment concentration is then calculated as $C(t) = T(t)/Q(t)$, and yield is the time integral of transport, as in (6). For a specified storm hydrograph $Q(t)$ over (t_0, t_1) and an initial supply distribution $S_i(t_0)$, $i = 1, \cdots, N$, (10) can be solved numerically for model time series of sediment supply, yield, and concentration.

The distributed model represented by (10) is more speculative than the single-compartment model of (4) and (5). For example, we have no compelling reason to write the transport curve term in (10) as aQ^b rather than aQ_i^b. In addition, the parameters of (10) are no longer easily interpreted or estimated. Here, for example, the concentration ratio R_c does not have a direct relationship to (r/S_0), as was the case in (4) and (5). In fact, each supply compartment realistically has its own depletion characteristics, that is, its own values a_i, b_i, p_i, and r_i. However, (10) was defined for lumped, rather than distributed, parameters in order to keep the number of parameters relatively small.

The distributed supply model was applied to the four Flynn Creek storm events, using the same calibration procedure and performance measure as with the single-compartment model. The total initial supply was set at $S_T(t_0) = 303$ tonnes, as in the single-supply model, and this sediment was distributed equally among all compartments at the start of storm 1, 1977. All four model storm simulations used $a = 1.0$ and $b = 4.0$. The availability parameter r took the value 26.5 for the three 1977 model events and 9.6 for the 1979 event.

Parameter values for the 'best fit' distributed model simulations were quite close to the single-supply model values. Never-

theless, we found the distributed model to be much more responsive to changes in streamflow and sediment supply. For example, during storms 2 and 3 in 1977 (see Figures 4 and 7), the distributed model did a better job of reproducing the steepness of observed concentration peaks. In the distributed model, new supply compartments were progressively accessed as streamflow increased and sediment transport increased more rapidly than could be simulated by (4) and (5). On the falling limb of the hydrograph, the reverse process occurs whereby fewer and fewer storage locations are accessed. The distributed model fits also had smaller concentration errors than the single-supply model, for three of the four storm events (Table 1). Of course, we had greater freedom in fitting the distributed model to storms 2 and 3, 1977, through the use of the distributed inputs, I_i.

Figure 6b illustrates the distributed supplies at three different times in the modeled 1977 event sequence. At the beginning of storm 1, all compartments contained $S_i(t_0) = S_T(t_0)/N$ tonnes. By the end of storm 1, about 10% of the total supply was removed, all from the first four compartments. For purposes of illustration, the volumes of sediment depleted from and added to the total supply shown in Figure 6b have been exaggerated slightly. In the model, Q_4 was chosen greater than 1.5 m^3 s^{-1}, the peak flow of storm 1, so compartments 5 and 6 were not accessed by the event. Between storms 1 and 2, we adjusted values of I_1, I_2, I_3, and I_4 until the fit shown in Figure 7 was achieved. The total of the four inputs was about half as large as the total yield from storm 1. Calibration of storm 3 followed the same procedure.

In short, the distributed model appeared to perform slightly better than the single-supply model. The distributed model's most important advantage, however, is that it provides for the eventual use of direct storage estimates from several sites, each of which is accessible for a different discharge level.

Application to a Controlled Reservoir Release

A unique set of suspended sediment data from Huntington Creek in central Utah provided us with a valuable test of the single-supply model. The creek is dammed, and a controlled release of water from Electric Lake Reservoir in August of 1979
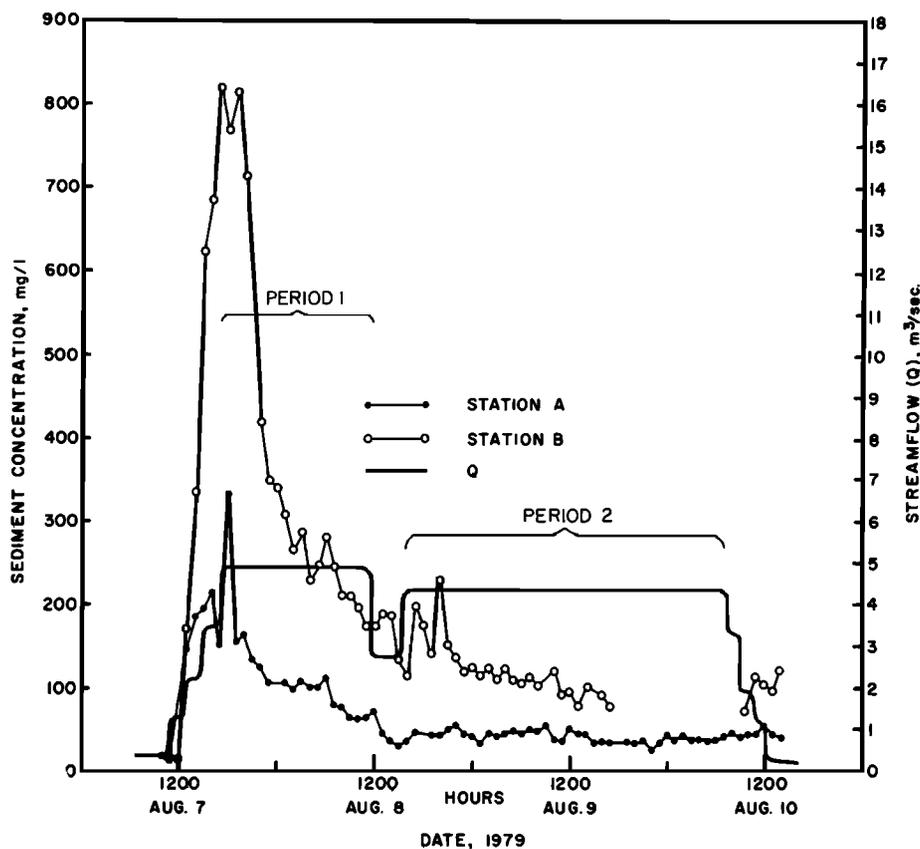
Fig. 8. Suspended sediment concentration and discharge during a controlled reservoir release, Huntington Creek, Utah, 1979.

produced two extended periods of high, constant streamflow, following a year of low-flow conditions.

During the flow release, bedload and suspended load transport were monitored by taking hourly samples at two stations. The methods and results of the study are reported by Beschta et al. [1981b]. Figure 8 presents the time series of discharge and suspended sediment concentration at the two stations. Station A was 1.4 km downstream of the dam, while station B was 6.7 km from the dam. In this reach of Huntington Creek, the mean gradient is 0.01 m/m, and the bankful width is about 6 m at station A and 10 m at station B. The channel bed material is mainly sand and fine gravel armored with material of median size = 20 mm. The armor layer was not disrupted by the high flow of the controlled release. The reservoir release was conducted during a period of clear weather, so suspended sediment source areas lay entirely within the channel.

We were especially interested in applying the model to the two time intervals labeled period 1 and period 2 in Figure 8. During period 1, stream discharge was constant at 4.9 m³ s⁻¹ for 20 hours, and the second period had a constant discharge of 4.4 m³ s⁻¹ for 41 hours. Beschta et al. [1981b] suggested that the decreasing sediment concentrations during these periods were the result of sediment supply depletion. In addition they noted that station B had about 5 times the length of upstream channel as a sediment source area, compared with station A; this almost certainly accounted for the much higher concentrations seen at station B.

These authors also examined linear regressions of (log C) versus (time) for the two periods at the two stations. They concluded that concentrations were decaying exponentially

over time during these periods, but the rate of decay during period 2 was considerably less than that during period 1 at both stations. Their hypothesis was that the drop in discharge from 4.9 to 4.4 m³ s⁻¹ between the periods was responsible for the lower decay rate in the second period.

We could have applied the distributed model to this event. However, if discharge remains constant, then the set of accessible supply compartments remains fixed over time. For analytical ease, we chose instead to view the Huntington Creek supply at each station as a single compartment containing all sediment available to flows of 4.9 m³ s⁻¹ during period 1 and 4.4 m³ s⁻¹ during period 2.

The single-supply model of (4) and (5) can be solved analytically if Q is constant. The solution leads to a formula for sediment concentration of the form $C(t) = 1/(k_1 + k_2 t)$, where the constants k_1 and k_2 are complicated functions of model parameters and initial conditions. Rather than explore this analytical solution further, we instead used the Huntington Creek data to compare the supply-based model with an exponential decay model for $C(t)$. The exponential decay model, which has the form $C(t) = C(0) \exp[-\alpha t]$, was used by Beschta et al. [1981b] to describe the concentration decreases in Figure 8. In addition, the exponential decay model would result if we had used a linear function for $g(S)$ in the single-supply model, rather than the exponential form (3).

The relationship between the single-supply model and exponential decay is not hard to uncover. Differentiation of (2) results in

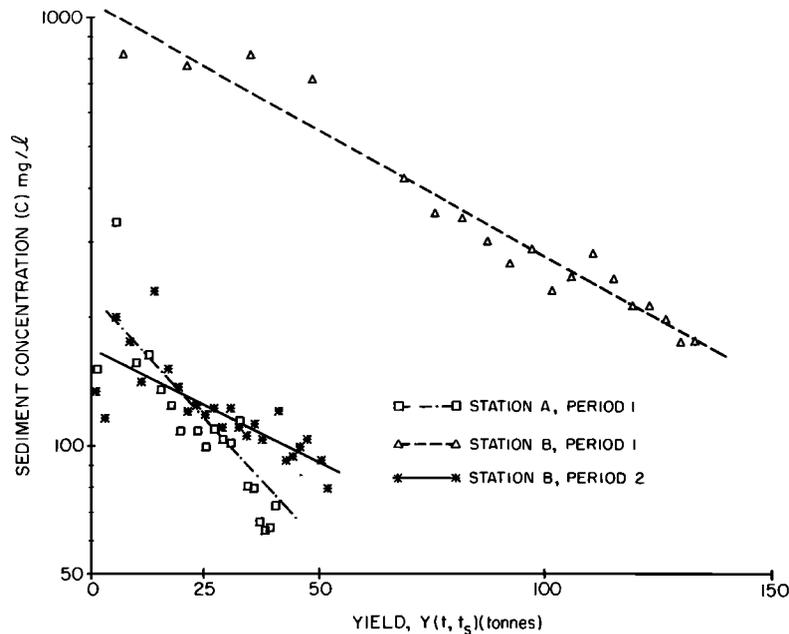$$\frac{dC}{dt} = aQ^b \cdot \frac{dg}{dS} \cdot \frac{dS(t)}{dt} \tag{13}$$

Fig. 9. Sediment concentration versus cumulative yield during 1979 Huntington Creek reservoir release.

Substitution of (4) yields

$$\frac{dC}{dt} = -aQ^b \cdot \frac{dg}{dS} \cdot C \cdot Q \qquad (14)$$

If we now take (3) for $g(S)$, then (14) can be written as

$$\frac{dC}{dt} = -K \cdot S \cdot C \qquad (15)$$

where $K = r \cdot a \cdot Q^{b+1}/S_0$ is a constant, for constant Q.

Thus the supply-based model implies that $C(t)$ decays exponentially, for constant Q, with a decay rate $\alpha = (K \cdot S)$ which is time varying. This result agrees with Beschta et al.'s [1981b] observation that the rates of decay were significantly reduced during period 2. Equation (15) shows that the reduced discharge Q could have resulted in the lowered decay rates seen in period 2. However, it is clear that a decreasing sediment supply $S(t)$ would also reduce the concentration decay rate. In any case, if the simple exponential decay model requires a time-varying rate parameter α, then it becomes less attractive as a predictive tool.

For a quantitative test of the supply-based model, we used a model equation which, like the case of simple exponential decay, has only one parameter to be estimated from the data. This simplification was possible because Q was constant. Recall that (9) relates sediment concentration to sediment yield for two instants in time having the same Q. Let t_s be the start of a period of constant discharge. Then (7) and (9) can be combined to give:

$$\log_e C(t) = \log_e C(t_s) - \frac{r}{S_0} \cdot Y(t, t_s) \qquad (16)$$

where t is any time after t_s for which Q remains constant.

In (16) the value of r/S_0 can be estimated from a regression of $\log_e C(t)$ on $Y(t, t_s)$. Figure 9 shows three such regressions based on the concentration data from stations A and B during period 1 and from station B during period 2. The concentrations from station A during period 2 were not included because they

appeared to have dropped to a level at which further decreases were lost in the background noise level. Beschta et al. [1981b] found that the slope of the $[\log_e C(t)]$ versus (t) regression was virtually zero during this period at station A. To confirm this, we calculated a rank correlation coefficient between $C(t)$ and t for the same data. The resulting value of Spearman's rho was negative, but it was not different from zero at a 5% significance level [Snedecor and Cochran, 1967]. Since Y is a cumulative variable, we could not use correlation coefficients from the regressions to test the validity of (16), but each data set appears to be scattered fairly evenly about its regression line.

The most interesting feature of the regressions in Figure 9 is their slopes. According to (16), each slope is an estimate of $(-r/S_0)$ for that particular data set. For station B, period 1, the estimate was $(r/S_0) = 0.007$ tonne^{-1}, and for period 2 at the same station, $(r/S_0) = 0.006$ tonne^{-1}. At station A in period 1, $(r/S_0) = 0.015$ tonne^{-1}. Since station A had a smaller sediment source area than station B, it should also have a smaller value of S_0. This hypothesis is supported by the relative sizes of (r/S_0) at the two stations; the station A estimate is about twice as large as the station B estimates.

It is also useful to compare the results from periods 1 and 2 at station B. It appears that (16) describes $C(t)$ with a single parameter value, (r/S_0), which is virtually unchanged over the two periods, in contrast to the varying exponential decay parameter α. Beschta et al. [1981b] reported that the value of α in period 2 at station B was about one half of its value during period 1.

As a direct, quantitative comparison of the supply-based model and the exponential decay model, we performed the following test: both models were used to predict $C(t)$ during period 2 at station B, based on parameter values estimated during period 1. For the exponential decay model, $C(t) = C(0)$ exp $[-\alpha t]$, a regression of $\log_e C(t)$ on t gave a value of $\alpha = 0.083$ hour^{-1} during period 1. For the supply-based model (16) we used the period 1 estimate of $r/S_0 = 0.007$ tonne^{-1}.

Equation (16) relates yield and concentration, but we need an expression for C as a function of time. Imagine (16) applied over

the short time interval $(t, t + \Delta t)$. If Δt is small, then we can approximate $Y(t + \Delta t, t)$ by $C(t) \cdot Q \cdot \Delta t$, as suggested by (6). With this approximation, (16) takes the form of a difference equation:

$$\log_e C(t + \Delta t) = \log_e C(t) - (r/S_0) \cdot Q \cdot C(t) \cdot \Delta t \qquad (17)$$

For a given value $C(0)$, (17) can be solved iteratively to give $C(\Delta t), C(2\Delta t), C(3\Delta t)$, etc.

Both models require a specified value of $C(0)$ as an initial condition. Figure 8 shows that during the first few hours of period 2, the concentration at station B varied erratically, probably because of the sudden increase in discharge. Accordingly, in both models we set $C(0) = 156$ mg l^{-1}, which was the average of $C(t)$ over the first 4 hours of period 2. We then set the time origin at hour 2 of the period and produced hourly predictions from both models for hours 5 through 25. Equation (17) used $\Delta t = 1$ hour and also included appropriate conversion factors (e.g., Δt is 1 hour, but Q has units of cubic meters per second).

Both predictions were compared with $C(t)$ data during period 2 by means of the same error criterion as employed in the Flynn Creek example. The average absolute concentration error, as defined in Table 1, was 59.2 mg l^{-1} for the simple exponential decay model but only 18.9 mg l^{-1} for (17), based on 21 hours of data. The exponential model seriously underpredicted $C(t)$ throughout the second period, and (17) overpredicted $C(t)$, but with only $\frac{1}{3}$ as large an error.

In summary, we found that the supply-based model usefully described the sediment concentrations which were observed following the controlled reservoir release. Estimated values of (r/S_0) were consistent with the relative size of sediment source areas for the two sampling stations. In addition, for period 2 at station B, the supply-based model produced more accurate concentration predictions than did a simple exponential decay model. The exponential decay model results if one uses a linear function $g(S) = \beta \cdot S$ in (14). Thus the station B predictions supported our choice of (3), rather than the linear function, to relate sediment concentration and sediment supply.

Discussion

Equations (4), (5), and (10) are useful models of sediment transport as a function of streamflow and sediment supplies. The models are mathematically simple, have few parameters, and can be calibrated to fit observed (C, Q) time series. Both models reproduce the sediment concentration dynamics of storm hysteresis and of seasonal decline. As a result, on a storm-by-storm basis, they fit observed concentration levels more closely than a sediment transport curve.

We feel that the supply-based approach shows promise as a predictive tool, and as a conceptual tool, in studying stream sediment yields. Of the two models, the single-supply version has the greatest predictive potential. We have not yet made a strong statistical case for the predictive superiority of the single-supply model over the transport curve (1). The transport curve has a few more degrees of freedom than the supply-based model. However, in some basins it may be possible to estimate independently two of the supply-based parameters, r and S_0, from channel surveys.

To make effective predictions with the single-supply model, we need to know the frequency and number of (C, Q) observations required for calibration. The Flynn Creek simulations indicate that good parameter estimates can be made from (C, Q) regressions on the rising hydrograph limb of the first fall

event and from sample values of R_c taken during the event. Further, the simulations suggested that the initial parameter values were relatively stable over time. Detailed sampling of later storms would not be necessary but could improve subsequent predictions. When new data become available, model parameters can be quickly updated; although our calibrations were done by trial and error, the model should lend itself well to on-line nonlinear programing methods for estimation of its few parameters.

The distributed supply model, on the other hand, can be a useful conceptual aid to sediment budget analyses. It suggests that identifiable sediment storage sites in a given channel and floodplain could be categorized by their accessibility to different discharge levels. If this categorization can be made, then the model could be used to help follow, over time, the depletion and resupply of specific sites.

A more theoretical approach to the problem of identifying storage locations could be taken by drawing on studies of the hydraulic geometry of river channels [e.g., Leopold et al., 1964; Yang et al., 1981]. Much of their work has yielded empirical expression of parameters such as channel width and depth as power functions of discharge. The distributed supply model could use hydraulic geometry relations to test various assumptions about the relative availability of sediment at various flow regimes.

It should be clear that the supply-based approach need not apply only to streams with storm event type hydrographs. If model parameters can indeed be estimated from a single runoff event, then the models may also be valuable for watersheds with snowmelt hydrographs. Regardless of the specific application, however, the most uncertain model features will continue to be the specific locations, sizes, and relative accessibility of sediment storage sites and their ultimate sources. We believe that a quantitative knowledge of sediment supplies is the key to improved modeling of sediment yield.

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